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MEASUREMENT OF THE LENGTHS OF DIGITIZED CURVED LINES

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ABSTRACT

An examination is made of methods of measuring the lengths of arbitrarily shaped smooth curves from their quantized representations, both in the absence and in the presence of noise. For 4-way encoded curves in the absence of noise, the length of the underlying smooth curve can be accurately assessed as a constant multiplied into n, the number of direction vectors in the curve, or as a function of n and the number of corners in the curve. Good measurements can also be obtained in the presence or absence of noise by means of an m-step polygon measure, or after direction or curvature smoothing. The methods are explained and their merits are compared.

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1. INTRODUCTION

Suppose that in a two-dimensional field of view there is a real object with a hard smooth boundary well differentiated from the background. Observers would agree that the object covers some definite area (A_0, say) and has a boundary of definite length (S_0, say). If the field of view, and the object on it, are quantized for the purpose of analysis, the resulting values of area and boundary length will differ somewhat from A_0 and S_0. If, in addition, the physical device which effects the transition from field of view to matrix of numbers is subject to random noise (and this is often the case), then the recorded value of boundary length, S_r, say, will further depart from the "true" value of S_0. Indeed, if the boundary could be corrupted by random noise of limited amplitude but of indefinitely high spatial frequency, S_r would tend to infinity, although in practice the highest noise frequency is determined by the grid spacing of the quantization.

In the accompanying paper (1) we examined the errors occurring when a straight line of known length is measured by means of its encoded representation. The same type of analysis could be used to determine the average error in measuring the length of a quantized form of a mathematical curve such as a circle or other conic section (cf.(2)). However, typical natural objects are not bounded by mathematical curves and in the computer analysis of visual scenes the task often arises of measuring the length of a curve of unspecifiable shape. This paper is concerned with the problem: given the recorded boundary list of a quantized closed curve, estimate the true length, S_0, of the original smooth curve.

We distinguish, for the purpose of discussion, two classes of curves: noise-free curves for which error in the encoded representation is due only to the quantization effect; and noisy curves for which the quantization error is compounded by the addition of random noise. In order to analyze noisy curves it is necessary to introduce some form of smoothing operation to diminish the effect of the noise. From among the many smoothing operations available, we have in the present context examined only the two which we consider simplest--curvature smoothing (3 - 6) and direction smoothing. However, whereas length determination involving smoothing operations can be used on noise-free curves, some simple methods of measuring lengths of noise-free curves fail in the presence of noise, as might be expected.

2. THE SPECIFICATION AND MEASUREMENT OF AREAS AND BOUNDARIES

If a continuous region of a picture is quantized onto a square grid, where the region, of area A_0, is well contrasted to the background, the region is represented by a set of picture elements (pixels), each of which is the same area and is specified by a co-ordinate pair (x_i,y_i) where $i = 1, 2, \ldots, t$ and t is the total number of pixels. The area, A_r, of the quantized region is evidently equal to $t \times$ (pixel area), or $A_r = t$ if the pixel area is unity.

Kulpa (7), and also Sankar and Krishnamurthy (8), have argued that the area enclosed by a digitized curve should be evaluated by use of Pick's theorem (9) which states that the area, A, of a polygon, the vertices of which lie on the points of a grid with unit cell of unit area, is given by $A = i + b/2 - 1$, where i is the number of grid points interior to the curve and b is the number of grid points on

the boundary. However, in dealing with quantized regions, it is necessary to be clear as to what is taken as the boundary. It is common practice to take as the boundary either $\{(x_j,y_j)\}$, a subset of $\{(x_i,y_i)\}$, containing the co-ordinate pairs of all those pixels which lie on the edge of the region, or $\{(x_k,y_k)\}$, the co-ordinate pairs of all those pixels which lie just outside the region (7). The implication of this practice is that the pixel coordinate pairs specify grid points; but it would accord better with the technology of picture processing to consider the pixel co-ordinate pairs to specify the centers of unit cells of the grid (Fig.1), in which case they can be referred to as lattice points. We can thus distinguish the grid point boundary, b_g, and the lattice point boundary, b_ℓ, together with the respective areas specified by them. This matter is considered in more detail elsewhere (10) where it is proved that Pick's formula applied to b_g gives an area equal to t, the total number of lattice points (i.e. the total number of pixels), and also that $b_g = b_\ell + 4$.

In the present work we have taken the area of a region to be the total number of pixels comprising it and its boundary to be the grid point boundary as illustrated in Fig. 1, chain-coded by a 4-way code (1).

An important matter to be noted at this point is that whereas the geometric length, S_r, of either the grid-point or the lattice-point boundary is a poor approximation to the true length, S_0, of the original unquantized boundary, the quantized area, A_r, is a good approximation to the unquantized area, A_0, provided that the region can be represented by several hundred or more pixels and that most

of the region is more than a few grid units across. This is illustrated in the results given below, but can be seen intuitively by considering the effect of noise. If noise is represented, as below, by pixels being randomly added to or subtracted from the boundary, the area within the boundary, A_r, will be unchanged, provided that the additions and subtractions are equal in number; but the length, S_r, of the boundary will be significantly increased.

3. METHODS OF MEASURING THE LENGTH OF ARBITRARY CLOSED CURVES

Circularity methods

We have included two methods of approximating the length of boundary of an irregularly shaped area since their results were easy to compute from our data. The deficiencies of the methods are, however, obvious and little attention is given to them in the results we quote below.

Circumference of equal-area circle. This method, self-explanatory, will clearly give a reasonably accurate estimate of the length of boundary of nearly-circular areas without important clefts or protuberances, but will fail otherwise. It has been used in a few cases for the measurement of appropriate objects (11).

Circumference of average-diameter circle. The average value, $\hat{d}$, is taken of the projection of the area onto lines at various directions; the length of boundary of the area is then estimated as $2\pi\hat{d}$ (12). In using this method we typically took directions inclined to the x-axis at 10 angles between 0° and 180°. The method works quite well for compact areas bounded for the most part by convex arcs, but fails

if there are significant re-entrant regions in the curve.

Methods not involving smoothing to eliminate noise

n_{cor}, correction to the number of boundary chain links. We have shown in the accompanying paper (1) that if an arbitrarily-oriented straight line of length ℓ is 4-way encoded in a sequence of n points the length n of the digitized line gives an average relative error $\bar{\varepsilon}_4 = \frac{n - \ell}{\ell} = \frac{4}{\pi} - 1$. It follows from this result that if the length of the grid unit were taken as $\frac{\pi}{4}$ instead of 1, the average relative error would be zero; but this method is rejected as a means of measuring the lengths of straight lines since it involves too large a standard deviation, implying that the relative error would be unacceptably high in the measurement of the lengths of lines inclined at certain angles to the axes. An arbitrarily shaped curve, however, may be regarded as the envelope of a series of tangents to it, inclined at a range of angles to the axes; and in the case of a closed continuous curve, there are tangents inclined at all angles between 0° and 360°. Hence, insofar as a 4-way encoded closed curve can be regarded as constituted of small segments of successive tangents, a summation of these segments will involve lines at all angles to the axes and the measured length obtained directly from the encoding will be exactly $(1 + \bar{\varepsilon}_4)$ times the true length. According to this argument, if an arbitrary continuous closed curve is 4-way encoded as n points or direction vectors, the true length of the curve is given by $n_{cor} = \frac{n\pi}{4}$.

c, corner count correction. We have shown (1) that for a 4-way encoded straight line the average relative error of length measurement

may be reduced to zero and the standard deviation to a tolerable level if the grid steps are taken as being of length $\frac{\pi(1 + \sqrt{2})}{8}$ (= 0.948) instead of 1 and if also a deduction of $\frac{\pi}{8\sqrt{2}}$ (= 0.278) is made for each corner encountered in the quantized line. Applying the same argument as that used to obtain n_{cor}, we have an alternative measure of the true length of the 4-way encoded arbitrary continuous closed curve: $c = 0.948n - 0.278k$ where k is the number of corners in the quantized curve.

p_m, the m-step polygonal boundary. In our account of the measurement of the lengths of straight lines (1) we considered the reduction in relative error which could be brought about if the 4-way encoded data were m-sampled; that is, the length of the line is taken as the sum of the lengths of vectors such as that between the points s_j and s_{j+m}. This method is here also applied to 4-way encoded arbitrary closed curves. We take $p_m = (\sum_{i=o}^{n} d_i)/m$ where d_i is the m-vector from s_i.

Methods designed to smooth out noise

Curvature smoothing. Since several techniques of curvature smoothing are of proven utility and are incorporated into procedures of picture processing (3 - 6) we have used it for the present purpose. For a continuous differentiable curve, the curvature at a point is defined as $\frac{d\psi}{ds}$ where ψ is the angle between the abscissa and the tangent to the curve at that point and s is the distance along the curve to the point from an arbitrary zero. (It is to be noted that for simple closed curves which do not loop over themselves, $\oint d\psi = 2\pi$.) For such a curve which is closed, the area A_0 enclosed by it is given by

$$A_o = \frac{1}{2}\ (\oint x dy - \oint y dx).$$

Hence
$$\frac{dA_o}{ds} = \frac{1}{2}\frac{d}{dy}\ (\oint x dy)\frac{dy}{ds}\ - \frac{1}{2}\frac{d}{dx}\ (\oint y dx)\ \frac{dx}{ds}$$

$$= \frac{1}{2}\ (x\sin\psi - y\cos\psi) \qquad (1)$$

since $\frac{dy}{ds} = \sin\psi$ and $\frac{dx}{ds} = \cos\psi$ from which, also, $y = \int_o \sin\psi ds$ and $x = \int_o \cos\psi ds$. Integration of eq(1) then gives

$$A_o = \frac{1}{2}\oint \sin\psi(\int_o^s \cos\psi ds)ds - \frac{1}{2}\oint \cos\psi(\int_o^s \sin\psi ds)ds \qquad (2)$$

The corresponding formula for a curve quantized into n elements of equal length δ_s, enclosing an area A_r, is

$$A_r = \frac{1}{2}\sum_{j=1}^{n} \sin\psi_j\left[\sum_{i=1}^{j} \cos\psi_i \cdot \delta_s\right]\cdot\delta_s - \frac{1}{2}\sum_{j=1}^{n} \cos\psi_j\left[\sum_{i=1}^{j} \sin\psi_i \cdot \delta_s\right]\cdot\delta_s \qquad (3).$$

For a 4-way-encoded curve, when $\delta_s = 1$ and ψ_i, the angle which the ith chain vector makes with the x-axis, has the value $\frac{k\pi}{2}$ (k = 0, 1, 2 or 3), it is readily shown that eq(3) gives the area within the grid point boundary as discussed above.

For the quantized curve, the curvature at a point is taken to be $\frac{\Delta\psi_d}{\delta_s}$ where ψ_d is the angle between a direction vector and the x-axis and $\Delta\psi_d$ is the difference between the values of ψ_d of the vectors leading to and departing from the point in question (3); note that true values of angles are used rather than coded values as in Eccles et al. (5). We have smoothed the curvature by the multiple convolution method with a rectangular window discussed by Eccles et al. (5); that is, for a window width w, the curvature at each point is replaced by an average of the curvatures of the $\frac{(w - 1)}{2}$ neighbours on each side of the point together with the value at the point itself, this

procedure being repeated a number of times.

The effect of curvature smoothing is to remove the influence of quantization error and noise from encoded curves(5). However, removing the influence of noise is equivalent, in effect, to removing or replacing an unknown number of points in the boundary list which originally comprises n points. Since it is not possible to know which points are to be removed or added, the matter is dealt with in the following way. After q convolutions, all n line elements are considered to be of equal length, though displaced from their original positions; the length of each element is chosen so that the area $A_r^{(q)}$, calculated by eq(3) after q convolutions, equals $A_r^{(0)}$, which is the best estimate available of A_0, the area enclosed by the unquantized curve, according to the argument given in the previous section; and the length ℓ, of the original curve is measured as $\ell_m = n\delta_{s,cor}$ where $\delta_{s,cor}$, the corrected value of δ_s, is given by $\delta_{s,cor} = \sqrt{\dfrac{A_r^{(0)}}{A_r^{(q)}}}$, and $A_r^{(q)}$ and $A_r^{(0)}$ are both calculated with $\delta_s = 1$.

Direction smoothing. This method is based on the following informal argument. If a continuous curve is quantized and encoded as a chain of n direction vectors, the contribution each vector ought to make to the measured length of the curve is approximately equal to the length of its projection onto the tangent to the curve at the point where the perpendicular bisector of the vector cuts the curve. This is illustrated in Fig. 2. Let $\underline{d}$ be a direction vector and let $\underline{u}$ be a unit vector along the tangent corresponding to $\underline{d}$. Then the length of the projection of $\underline{d}$ onto the tangent is given by the scalar product $\underline{d}.\underline{u}$ and the length of the curve will be given by $\sum_{i=o}^{n} \underline{d}_i . \underline{u}_i$.

Since $\underline{d}$ and $\underline{u}$ are both unit vectors, it remains to determine only their directions to calculate $\underline{d}.\underline{u}$. The direction of $\underline{d}$ is known from the original encoding; the direction of $\underline{u}$ is taken to be an average direction of the quantized curve in the interval spanned by $\underline{d}$. The average we used was obtained by the convolution method described above for curvature smoothing, using one or more convolutions. As in the case of curvature smoothing, a small number of convolutions with a rectangular window serves to eliminate disturbances caused by noise in the boundary.

4. EXPERIMENTS AND RESULTS

Apparatus

Some experiments reported here were made using a Grinnell Systems Inc. graphics display connected on line to PDP11 computer at the Computer Vision Laboratory, University of Maryland and other results were obtained with the computer-controlled flying-spot microscope in the Biophysics Laboratory, Chelsea College (13), although this did not allow precise calibration as did the Grinnell system.

The Grinnell system presents a scene viewed by a conventional television camera as rectangular array of 512 x 480 pixels on a square grid, each point being displayed on an 8-bit grey level scale (i.e. providing 256 grey levels). To calibrate the system for the purpose of the present measurements of length, the television camera was set to view a sheet of graph paper at normal incidence. A simple computer program was written to determine the number of pixels along the straight line joining the co-ordinates of two cursors which can be independently positioned on the digitized picture by the trackball which is part of the system. It was then possible to check the linearity of all parts of the digitized picture. The aspect ratio of the television camera and the x- and y-deflection controls of the monitor screen on which the digitized picture is shown were all carefully adjusted until the number of pixels per unit length in any part of the scene viewed (i.e. the sheet of graph paper) departed by less than 1% from the average value. The discrepancies in the calibration ratio in fact occurred at the edges of the picture, so that in practice the object measured

was placed in the middle of the scene, in the same plane as had been calibrated with graph paper. Since the apparatus was also used for other purposes, it was recalibrated for each experiment.

<u>Measurement of the lengths of noise-free curves</u>

Using the Grinnell system, measurement were made of the lengths of plastic-covered wire twisted into arbitrary planar shapes. The color of the plastic covering was chosen to contrast sharply with the background (e.g. white-covered wire was viewed against black paper). A threshold was chosen by examining the thresholded picture to ensure that it was as free as possible of noise in the boundary of the wire. The threshold was then used in a computer programme which tracked the boundary, encoded it and calculated its length by the various methods discussed in the previous section. That the boundary was essentially noise-free was indirectly checked in a few cases by repeating the measurements with the threshold raised or lowered by a factor of more than 1.5. The recorded measurements showed that the resulting change in the number of direction vectors encoding the boundary and in the various measures of length was less than 0.5%; this contrasts with the measurements reported below in which random noise added to a smooth boundary increases the number of direction vectors needed in the encoding by amounts up to 90%.

In a first series of measurements, a length of wire was soldered into a loop, bent into arbitrary shapes (similar to that of Fig.3(a)) and measured (Table 1, first column). The length of wire was measured with a standard rule before soldering, with allowance made for the

soldering overlap; and after the measurements it was cut, straightened out and remeasured with the rule. However, the difference between the measured lengths of the inner and outer boundaries of the wire loop was appreciably greater than the difference between the measured lengths obtained by the reliable methods among those used. For the experiment recorded in the first column of Table 1, in which the averages are over measurements of the inner and outer boundaries of three different shapings of the wire loop, the difference in lengths of the inner and outer boundaries was, on average, 6.3%.

In a second set of experiments, pieces of wire of known length were measured unlooped. The boundary that was measured, after the wire had been twisted into a number of different shapes, was then twice the length of the wire plus twice its diameter. The results of measuring such a series are given in the second column of Table 1.

Measurement of the lengths of noisy curves

In view of the accuracy of some measurements of the lengths of curves which were free of noise, as demonstrated in Table 1, noisy curves were examined in the following way. A smooth version of a curve was created and its length measured several times to give n_{cor}, c and p_7 values, the average of all of which was taken to be the true length of the curve. A noisy version of the curve was then produced, enclosing the same area as the smooth version. Curvature smoothing and direction smoothing of the noisy curve were used to provide the measures cs(w,q) and ds (w,q) of the length of the curve after the effect of noise had been reduced by making q convolutions of a window of width w according to the procedures described previously. The values of w and q were

varied systematically to assess the amount of smoothing required to arrive at the presumed true length of the curve.

In one set of experiments the smooth curves were the boundaries of shapes cut cleanly out of paper and viewed by the Grinell system; examples are shown in Fig.3(a) and (b) which display the contents of the picture store. After measurement of the boundary of the area, the contents of the store were dealt with by a program which traced round the boundary and at each pixel either deleted it, added another in the outward direction or did nothing. The boundary was traced one or more times in this way and the action taken at each step was determined by a random number generator, used so that there was equal chance of adding or removing a pixel. The effect of this procedure on the shapes of Fig.3(a) and (b) is shown in Fig.3(c) and (d) in each of which there is nearly 100% of noise in the boundary, i.e. the numbers of points in the boundary of the noisy curves are nearly twice the corresponding numbers for the smooth curves.

In a second series of experiments, appropriate isolated objects, such as epidermal cells and muscle fibres in transverse section, were viewed in the flying spot microscope. The smooth curves were obtained by viewing the objects with the microscope slightly out of focus. Sharpening the focus then admitted higher spatial frequencies and so gave a real or seeming addition of noise to the boundary of the object. Fig. 4 shows the traced boundary, photographed from the monitor screen, of two overlapping human red blood cells as the microscope is brought into successive stages of sharper focus.

Boundaries with various amounts of noise were measured, the amount of noise being quantified as the extra proportion of direction vectors in the noisy boundary relative to the smooth one. In all, 18 sets of measurements were made on boundaries corrupted by up to 93% noise. The results of these measurements showed, broadly, that as the amount of noise in a boundary was increased the "true" length of the boundary could be obtained either by increasing the window width or number of convolutions in the curvature and direction smoothing methods, or by taking larger steps in the m-step polygon method.

The dependence of the calculated length of boundary on window width and number of convolutions is shown in Fig. 5 for both curvature smoothing and direction smoothing of a typical noisy curve. In this example the measured boundary was corrupted with 55.3% noise. It can be seen from the semilogarithmic plot that for both methods the calculated length falls steadily, but the rate of fall is slow after the first four or five convolutions. The low broad peak near the second convolution of the direction smoothing calculations was a consistent finding. The boundary length calculated by the m-step polygon method fell as m increased, in a manner similar to that detailed in Table 1 for noise-free curves. The two circularity measures, the circumferences of the same-area and average-diameter circles, were barely changed from the smooth boundary case and were as much in error (cf. Table 1). Both n_{cor} and c rose in proportion to the amount of noise in the boundary and were correspondingly in error.

As a guide to the accuracy of convenient procedures, Fig. 6 summarizes the results of calculations after curvature smoothing with two convolutions or direction smoothing with three convolutions. The graphs give the

probability, expressed as a percentage, that the calculated length will differ from the "true" length by not more than the indicated error. Measurements by the m-step polygon method gave results very close to those of direction smoothing on substituting step length for window width, i.e. on writing p_w for ds(w,3).

5. DISCUSSION

In considering the merits of the various methods of measuring the lengths of continuous curved lines which have been quantized for the purpose of computer analysis, we can immediately dismiss the two circularity methods, the circumferences of the same-area and average-diameter circles. Table 1 shows that both are seriously in error if there are re-entrants in the curve being measured, and neither would have meaning if the curve were not closed. In the following paragraphs we therefore confine ourselves to considering the three methods which do not involve explicit smoothing (correction of the number of direction vectors (n_{cor}), corner count correction (c), and the m-step polygon boundary (p_m)) and the two methods which do (curvature smoothing and direction smoothing). Furthermore, the discussion of curvature and direction smoothing is limited to the cases of 2 and 3 convolutions respectively of a rectangular window since these may be computationally simplified as one-pass operations with suitable shaped windows, triangular and cloche-shaped. Apart from the computational inconvenience of convolving many times, it has been shown (5) that a few convolutions with a rectangular window of width 5 or more is quite sufficient to smooth away typical noise disturbances.

Considering first the measurement of noise-free curves, Table 1 shows

how well the five methods agree. In the absence of noise, the lengths calculated by the curvature and direction smoothing methods change very slowly as window width is increased; and although not recorded in Table 1, the same is true as the number of convolutions is increased. For the noise-free curves, in comparing the lengths n_{cor}, c, p_5, p_7, cs(5,2), cs(7,2), ds(7,3) and ds(9,3), all the values are within 1.5% of the grand mean in each case given in Table 1. The grand mean of the eight values is, in turn, within a fraction of one percent of the value expected from the known length of the unquantized curve; and the mean of just the three quantities n_{cor}, c and p_7 comes even closer to the expected value.

The very close agreement between the results of the five methods implies that all are equally good at measuring the lengths of arbitrary noise-free closed curves; hence local convenience or simplicity of computation may be used as the criterion for choosing between them. On this basis, measurement by computation of n_{cor} is the most attractive method since the boundary or curve cannot be specified without the value of n, the number of direction vectors, being known. The next simplest method is the computation of c which is obtained from n and the number of corners. However, the measure obtained from n_{cor} gives good values only if tangents to the curves being measured have a fairly even distribution of inclinations in a range of angles which is an integral multiple of 45°. The value of n_{cor} would be seriously in error if used as a measure of length in, say, an engineering drawing consisting mostly of horizontal and vertical lines meeting in sharp corners. The measure obtained from c has the same restriction as that obtained from n_{cor}, but less severely.

The measure obtained from the m-step polygon method is more complicated to calculate than n_{cor} or c but has the merit that it can be applied to straight as well as to curved lines and to noisy as well as smooth ones. It is shown in the accompanying paper (1) that in measuring the lengths of arbitrarily inclined straight lines, the average error in the value of p_5 is 0.92% and in the value of p_7 is 0.47%. About the same amounts of error are found in the values of p_5 and p_7 for smooth curves (Table 1). As noise is introduced into the curve it becomes necessary to go to greater step lengths to obtain the "true" length of the curve. For a noisy curve, the probability of p_7 being within 1% of the "true" length is only about 10% and of being within 10% is about 65%; but p_{13} is virtually certain to be within 10% of the "true" value and has about a 45% probability of being within 3% of it.

In practice, one might in the course of analyzing a quantized picture encounter and be required to measure the length of a curve which was evidently noisy, yet was accompanied by no indication of how much noise it contained. From observation of many curves, we have found that one with 50% noise compared with its smooth counterpart looks very noisy indeed (Fig. 4). A curve with 100% noise in it (Fig.3) looks so ragged and "moth-eaten" that one might consider it to be qualitatively different from a merely noisy version of the underlying smooth curve. In view of these considerations, it is not possible to specify a measurement method which will assuredly provide an accurate value of the "true" length of a noisy curve; some judgement must be exercised by the experimenter in selecting the measurement method and the conditions of its use.

For noisy curves, the measures obtained from n_{cor} and c can be dismissed immediately as being in error directly in proportion to the amount of noise present. However, while the values obtained from the measures p_{13} and, as shown in Fig. 6, cs(11,2) or ds (13,3) are all virtually certain to be within 10% of the "true" value of curves with up to 75% noise, these values will be on the low side if there is not much noise present. In cases of low noise, the use of slightly narrower window width for curvature or direction smoothing or a shorter step length for the polygon would give a better measure.

It would be possible to draw a series of graphs, like those of Fig. 6, for each level of noise in a curve, but such a series would be of limited use since it is hardly possible to know in advance how much noise is contained in the curve being measured. Some estimate of the amount of noise could be obtained by comparing the original length of the noisy curve, measured by n_{cor} or c, with the length after smoothing, assuming that an appropriate amount of smoothing had been used. Such a calculation could lead to an approach to a measure of the "true" length of the curve by successive approximation, but our results do not indicate that the computational effort involved would justify the improvement in accuracy which would reduce the error from not more than 10% to, possibly, not more than 5%. If the "true" length is to be measured of an evidently noisy curve we recommend, therefore, using one of the measures p_{13}, cs(11,2) or ds (13,3) together with the interpretation given by Fig.6.

Which of the three measures is to be used would probably depend on other considerations. The curvature smoothing method cannot be used with open curves; but it may be the method of choice if smoothed curvature

is also to be used for segmenting the boundaries of conjugated objects (5). Direction smoothing can be used to measure the lengths of open curves and involves somewhat less computation than curvature smoothing, but uses more storage space since the computation requires the original direction as well as the smoothed direction. Both convolution methods have the advantage that analysis (5) provides knowledge of the spatial frequencies which are being eliminated by the smoothing procedure. The measure given by the m-step polygon is not supported by such an analysis, but it involves very much simpler computation than the other two methods and would be the method of choice if the requirement is to obtain a rapid measurement of the "true" length of a quantized curve corrupted with a relatively small amount of noise. It is also of interest that storing an m-sampled version of a boundary has the advantage of requiring less data storage space than does the original 4-way coded boundary (1); if such data were in any case being stored, the merit of using the p_m measure of "true" length of boundary would be even greater than otherwise.

Table 1. Measurements of the lengths of noise-free boundary curves

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EXPERIMENTAL CONDITIONS

	Closed loop of wire	Open length of wire
Length of wire (mm)	224.0	189.5
Diameter of wire (mm)	2.05	1.95
Calibration factor (pixels mm^{-1})	3.261	3.858
Length of boundary (mm)	224.0	383.0
Area covered by wire (mm^2)	459.2	369.5
Number of readings	6	7

DIRECTLY-OBTAINED DATA

	Closed loop of wire	Open length of wire
Average number of direction vectors in boundary, $\bar{n}$	932.3	1878.0
Length corresponding to $\bar{n}$ (mm)	285.9	486.8
Average measured area (mm^2)	465.8	362.6

Table 1 continued . . . .

Table 1 continued.

METHOD OF LENGTH MEASUREMENT (all values in mm)

Circularity methods		
Same-area circle	167.0	67.5
Average-diameter circle	200.6	165.8
Methods not involving smoothing by convolution		
Correction to number of direction vector, $\overline{n}_{cor}$	224.6 ± 3.6	382.0 ± 4.8
Corner count correction, $\overline{c}$	226.3 ± 3.3	383.7 ± 0.5
m-step polygon, $\overline{p}_3$	228.9	393.7
$\overline{p}_5$	225.5 ± 3.2	386.7 ± 1.0
$\overline{p}_7$	222.5 ± 7.8	384.5 ± 1.2
$\overline{p}_9$	221.6	383.1
$\overline{p}_{11}$	220.8	381.9
Methods which involve smoothing by convolution		
The expression (w,q) indicates a window of width w convolved q times		
Curvature smoothing, $\overline{cs}(5,2)$	228.9 ± 5.3	383.5 ± 10.1
$\overline{cs}(7,2)$	227.9 ± 5.3	383.1 ± 13.7
$\overline{cs}(9,2)$	226.9	383.2
$\overline{cs}(11,2)$	-	382.9
Direction smoothing, $\overline{ds}(5,3)$	224.8	381.9
$\overline{ds}(7.3)$	224.2 ± 8.2	380.4 ± 0.9
$\overline{ds}(9,3)$	224.0 ± 8.2	379.6 ± 0.9
$\overline{ds}(11,3)$	-	378.5

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Fig. 1. The points A, B, C. etc. are specified by pixel co-ordinates. These points can be regarded as determining a grid (drawn in broken lines) or as being lattice points lying within the cells of the grid determined by the grid points a, b, c, etc.

Fig. 2. A smooth boundary is digitized into a sequence of points which include a, b, c, etc. with unit grid spacing. The contribution required from each direction vector ($a \rightarrow b$, etc.) to the summation which will give the length of the curve is the projection of that vector onto its corresponding tangent to the original curve. Thus the contribution of (ab) is ($a'b'$), of (cd) is ($c'd'$), etc.

Fig. 3. (a) and (b) are printouts of two initial arbitrary shapes cut out of paper and viewed by the television camera of the Grinnell Systems Inc. graphics display. (c) and (d) are printouts of the same shapes after random noise has been added to their boundaries in the picture store.

Fig. 4. Boundaries, tracked in the automated flying spot microscope, of a pair of human red cells slightly overlapping each other. From left to right the microscope was brought into increasingly sharper focus. Taking the left-most (out-of-focus) curve as the smooth one, the amounts of noise in the others are, respectively, 29.4% and 171.9%

Fig. 5. Graphs of calculated lengths of the boundary of a noisy curve, similar to that shown in Fig. 3(c), after various amounts of smoothing. Curvature smoothing and direction smoothing were achieved by means of rectangular windows convolved several times with the noisy boundary. The "true" length of the curve was measured in the absence of noise, as explained in the text. The amount of noise in the measured boundary before smoothing was 55.3%

Fig. 6. Graphs summarizing the results of measurements of "true" lengths of noisy curves. The graphs give the probability of the measured length being within the error marked on the abscissa when the smoothing conditions are given by (w, q), w being the window width and q being the number of convolutions used. The corresponding curves for measurement by the m-step polygon method are indistinguishable from the direction smoothing curves if p_w is substituted for ds (w,3).

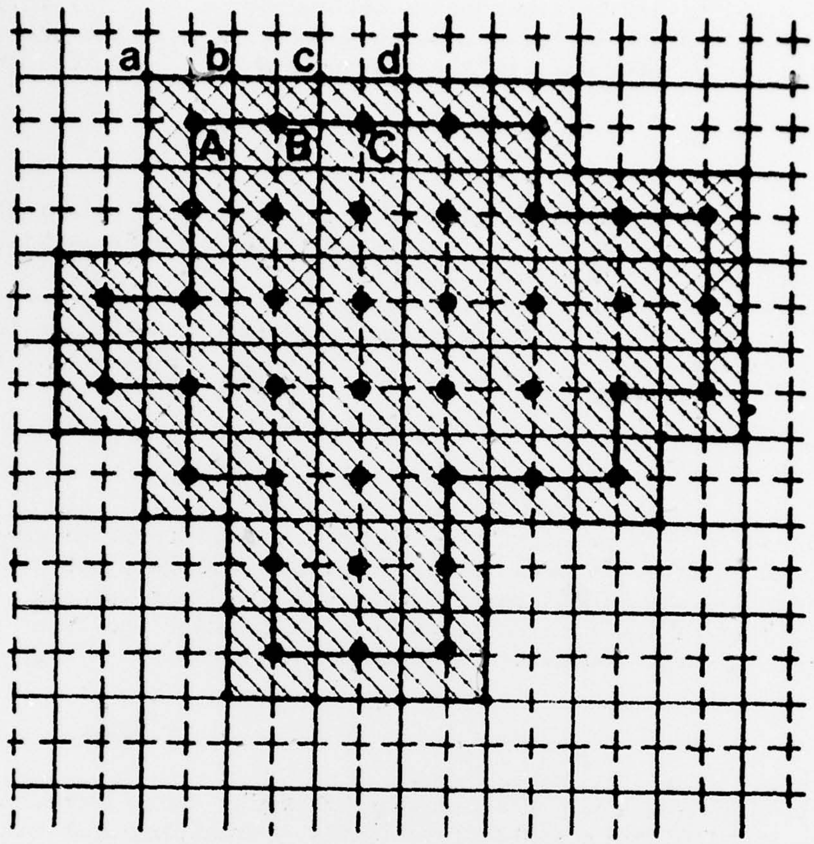

Figure 1

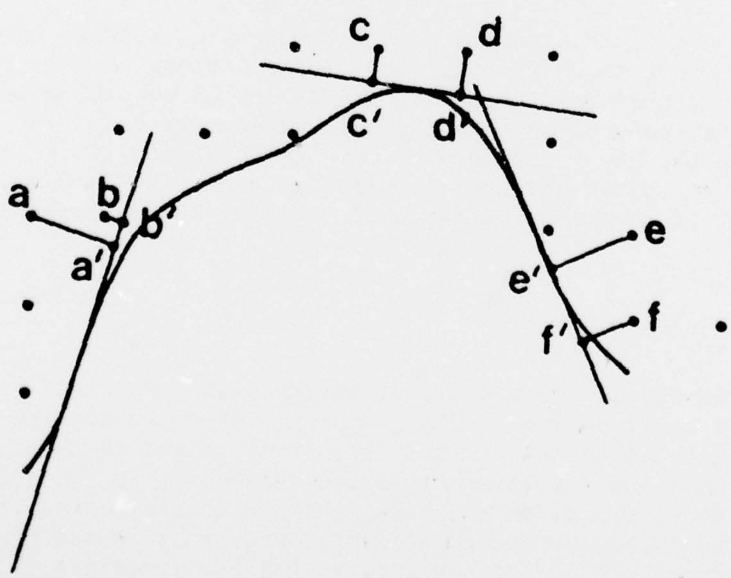

Figure 2

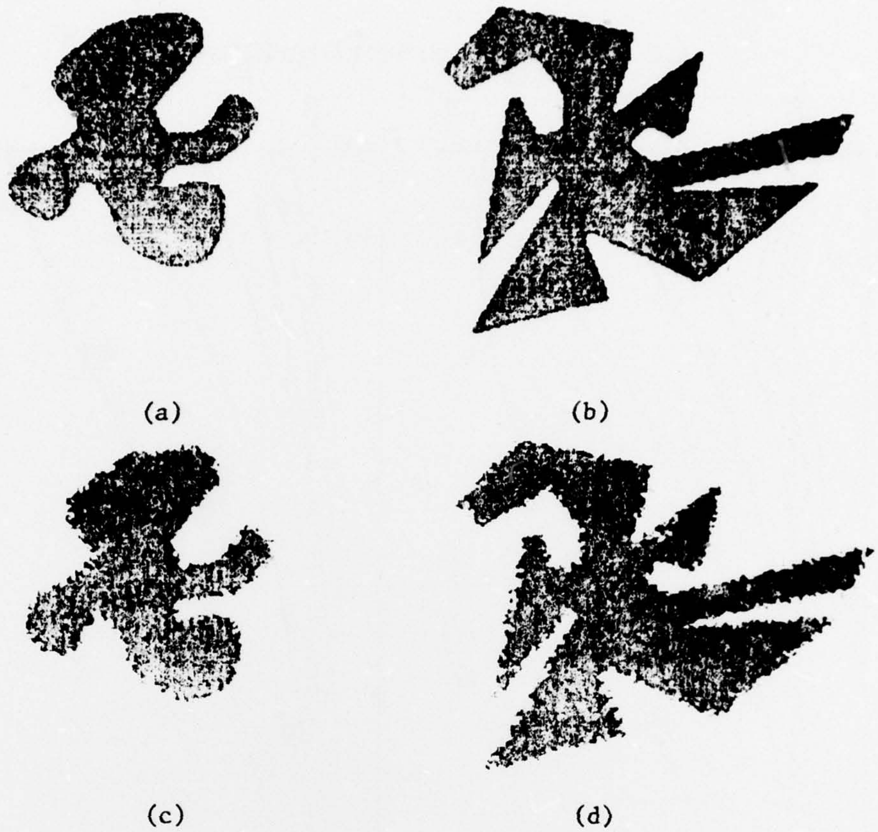

Figure 3

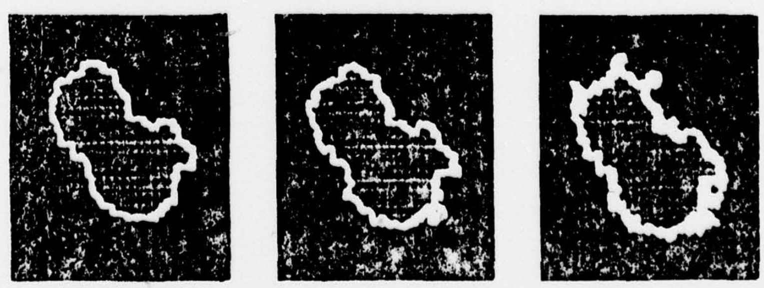

Figure 4

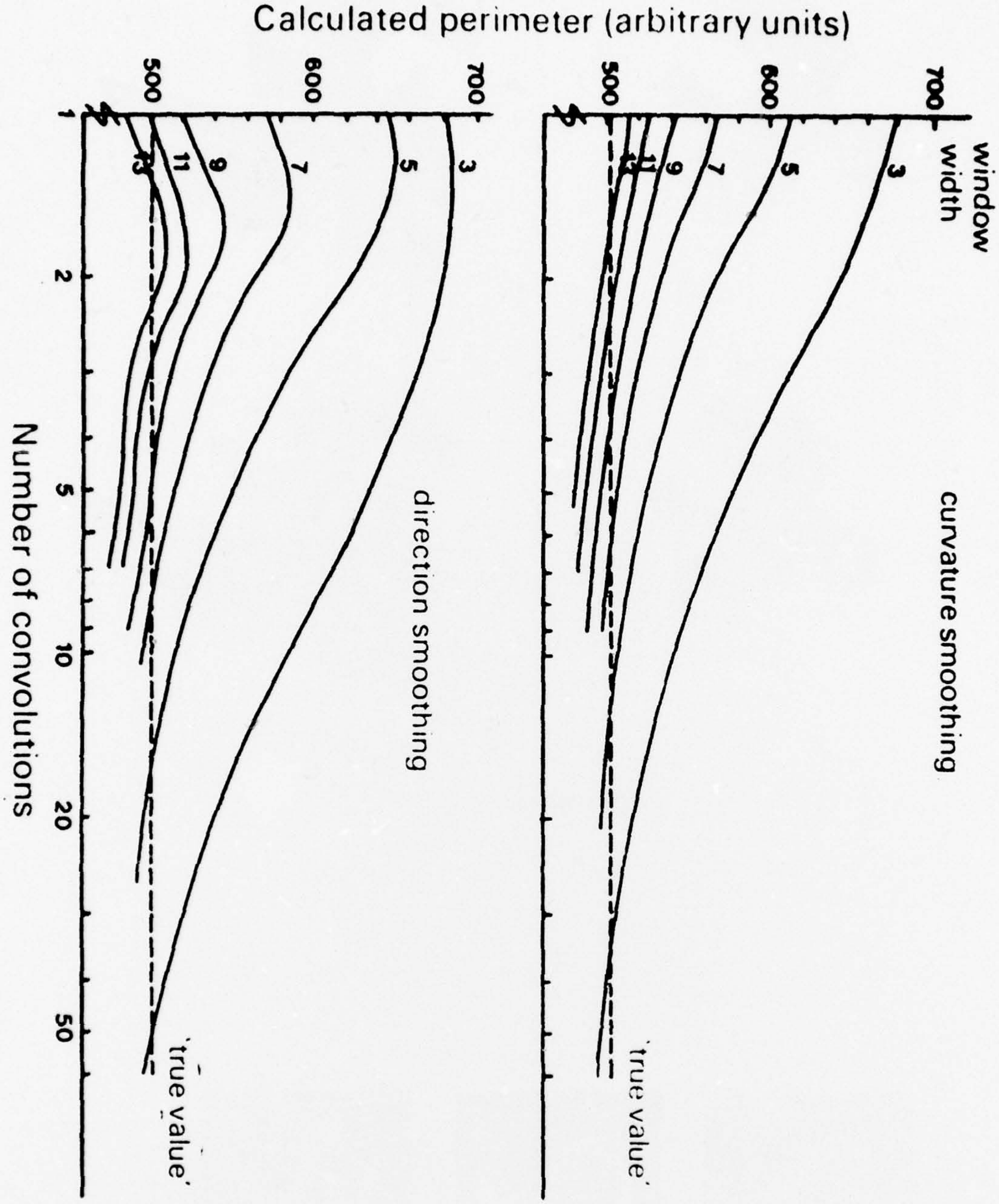
Calculated perimeter (arbitrary units)
700
600
500
window width
3
5
7
9
11
13
curvature smoothing
'true value'
direction smoothing
1
2
5
10
20
50
Number of convolutions

Figure 5

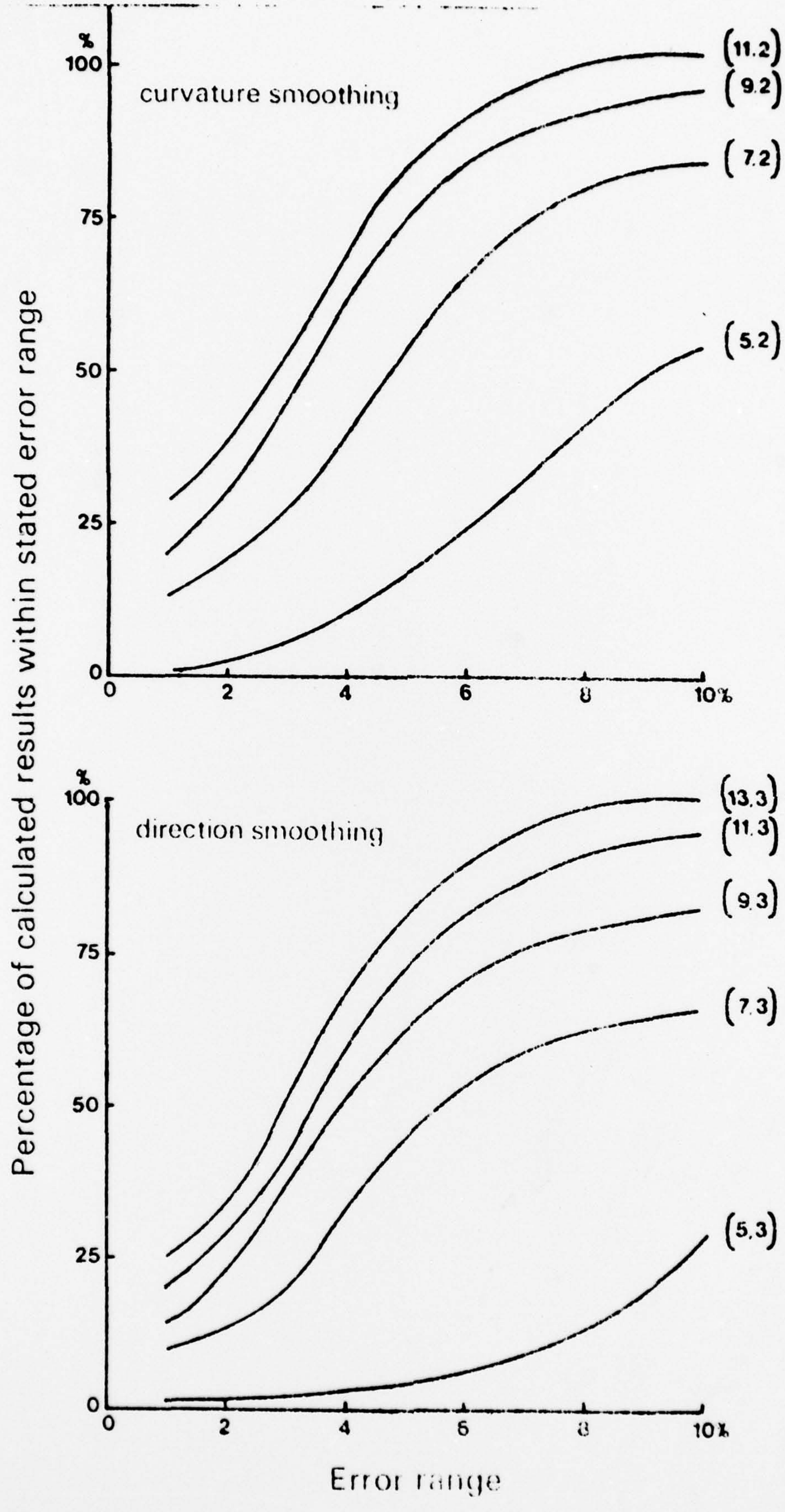
Percentage of calculated results within stated error range
curvature smoothing
%
100
75
50
25
0
0
2
4
6
8
10%
(11.2)
(9.2)
(7.2)
(5.2)
direction smoothing
%
100
75
50
25
0
0
2
4
6
8
10%
(13.3)
(11.3)
(9.3)
(7.3)
(5.3)
Error range

Figure 6

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An examination is made of methods of measuring the lengths of arbitrarily shaped smooth curves from their quantized representations, both in the absence and in the presence of noise. For 4-way encoded curves in the absence of noise, the length of the underlying smooth curve can be accurately assessed as a constant multiplied into n, the number of direction vectors in the curve, or as a function of n and the number of corners in...

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20 Abstract continued.

the curve. Good measurements can also be obtained in the presence or absence of noise by means of an m-step polygon measure, or after direction or curvature smoothing. The methods are explained and their merits are compared.